

Calculating more terms in Cloitre's sequence

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Abstract

We provide more terms of a sequence given by Cloitre. Previously the first 29 terms were known; we use a faster method of computing the sequence due to Jan Bütke to calculate the first 315 terms.

1 Introduction

Let $a = (a_n)_{n \geq 1}$ be the integer sequence inductively defined by

$$\begin{aligned} a_1 &:= 1 \\ a_n &:= |a_{n-1} - \gcd(a_{n-1}, n-1)| \end{aligned}$$

and let $b = (b_k)_{k \geq 1}$ the sequence of indices n for which a_n is zero. Cloitre defines the sequence b in [Clo, page 3] (see also [A186253](#) in the OEIS) and conjectures that all terms of b_k are prime and $b_k \sim c2^k$ when $k \rightarrow \infty$, for some constant $c = 1.186\dots$. In the OEIS-entry we can read $c = 1.1861\dots$. In an [Clo, APPENDIX 1], Cloitre supports this conjecture by giving the first 29 terms of the sequence b :

2, 5, 11, 23, 47, 79, 157, 313, 619, 1237, 2473, 4909, 9817, 19603, 39199, 78193, 156019, 311347, 622669, 1244149, 2487739, 4975111, 9950221, 19900399, 39800797, 79601461, 159202369, 318404629, 636788881.

The value $a_{n-1} - \gcd(a_{n-1}, n-1)$ becomes negative only if $a_{n-1} = 0$. We have $a_{b_k} = 0$ by definition and obtain $a_{b_k+1} = b_k$.

A straightforward method of calculating the sequence b consist of calculating the sequence a inductively and giving out the next term of b every time a_n is zero. This can for example be done in *pari* using the following line:

```
a=1; for(n=2, 10^20, a=abs(a-gcd(a, n-1)); if(a==0, print1(n, ", ")))
```

(see also the *pari* code giving in [Clo] and [A186253](#).)

When we let this run on a desktop PC for several weeks, we get a few more terms of b : 1273577761, 2547155419, 5094310069, 10188620041, 20377200079, 40754397121, 81508794229, 163017588457, 326034863503, 652069726981, 1304139453961, 2608278775139.

2 Jumping ahead

In this section we present a faster method of computing b . The idea and the code presented is by Jan Bütke.

We look at the sequence a . When a_n is zero, that is if $n = b_k$ for some k , for the term $n + 1 = b_k + 1$ we get $a_{n+1} = b_k$. If a_n is not zero and if $\gcd(a_{n+1}, n + 1) = 1$, we get $a_{n+1} = a_n - 1$. The sequence a only makes bigger jumps if $\gcd(a_{n+1}, n + 1) > 1$. The first of such jumps happens for the smallest m_1 such that $d_1 := \gcd(b_k - (m_1 - 1), b_k + m_1) > 1$. In fact we define inductively m_r to be the smallest positive integer such that

$$d_r := \gcd \left(b_k - \sum_{\ell=0}^r (m_\ell - 1) - \sum_{\ell=1}^{r-1} d_\ell, b_k + \sum_{\ell=1}^r m_\ell \right) > 1$$

To actually determine d_1 and m_1 we solve the following congruences:

$$\begin{aligned} b_k - (m_1 - 1) &= 0 & \text{mod } d_1 \\ b_k + m_1 &= 0 & \text{mod } d_1 \end{aligned}$$

Here the following fact comes in useful.

Fact 1. For integer A , B and odd d we have

$$d \mid A \text{ and } d \mid B \iff d \mid A + B \text{ and } d \mid A - B$$

By applying this fact we obtain:

$$\begin{aligned} 2b_k + 1 &= 0 & \text{mod } d_1 \\ 2m_1 - 1 &= 0 & \text{mod } d_1 \end{aligned}$$

We want to find the smallest positive m_1 that solves these congruences for some positive d_1 . This can effectively be done by considering the prime factors of $2b_k + 1$. Similarly in order to find inductively defined m_r and d_r we consider the system

$$\begin{aligned} 2b_k + 1 - \sum_{\ell=1}^{r-1} (d_\ell - 1) &= 0 & \text{mod } d_r \\ \sum_{\ell=1}^r 2(m_\ell - 1) + \sum_{\ell=1}^{r-1} d_\ell &= 0 & \text{mod } d_r \end{aligned}$$

Putting it all together gives the following *pari* code:

```
next_a(last_a) = {
  local(A=last_a,B=last_a,C=2*last_a+1);
  while(A>0,
    D=divisors(C);
    k1=10*D[2];
    for(j=2,matsize(D)[2],d=D[j];k=((A+1-B+d)/2)%d;
      if(k==0,k=d); if(k<=k1,k1=k;d1=d));
    if(k1-1+d1==A,B=B+1);
    A = max(A-(k1-1)-d1,0);
    B = B + k1;
    C = C - (d1 - 1);
  );
  return(B);
}
```

```
a=2
while(true,print1(a," ");a=next_a(a))
```

With this program we are able to calculate the first 300 terms of the sequence, given in a table in the appendix.

2.1 Another sequence

From the program it becomes clear, that we can define another sequence $(\alpha_n)_{n \geq 1}$, where α_n is the result of `next_a(n)`. In other words: α_n is the smallest index $i > n$ such that for the inductively defined sequence

$$\begin{aligned} a_n &:= 0 \\ a_i &:= a_{i-1} - \gcd(a_{i-1}, i-1) \quad \text{for } i > n \end{aligned}$$

we have $a_i = 0$. Let's write α as a function

$$\begin{aligned} \alpha &: \mathbb{N} \rightarrow \mathbb{N} \\ n &\mapsto \alpha(n) := \alpha_n \end{aligned}$$

Here are some observations for the function α .

- a) We have $b_n = \alpha^{n-1}(2)$, where α^{n-1} is α iterated $n-1$ times.
- b) $\alpha(13) = 21$
- c) $\alpha(i)$ is prime for all $0 < i \leq 10^9$ if $i \neq 13$.
- d) $\alpha(i) = 2i + 1$ if and only if $2i + 1$ is prime.
- e) $\alpha(i) = 2i - 1$ if and only if $2i - 1$ is a prime of the form $6z + 1$.
- f) $\alpha(i) = 2i - 3$ if and only if $2i - 3$ is a prime of the form $30z + 1$.
- g) $z := \alpha(i) = 2i - 5$ if and only if z is prime and one of the following three is the case.
 - I) $z = 0 \pmod 3$ and $z = 13$
 - II) $z = 1 \pmod 3$ and $z \not\equiv 0 \pmod 7$ and $z \not\equiv 0 \pmod 11$ and $z \not\equiv 0 \pmod 13$ and either
 - i) $z \not\equiv 4 \pmod 5$ and $z = 1 \pmod 7$ or
 - ii) $z = 4 \pmod 5$ and $z \not\equiv 9 \pmod 11$ and $z \not\equiv 11 \pmod 13$.
 - III) $z = 2 \pmod 3$ and $z \in \{8, 71, 92\} \pmod{105}$.

3 Asymptotics

As noticed by Cloitre, it seems like $b_n \sim c2^n$ for $c = 1.186\dots$. We attach a table of the values $c_n := \frac{b_n}{2^n}$. We plot the sequence absolute values of differences of c , i.e. $(|c_{n+1} - c_n|)_{n \geq 1}$. From this plot it looks plausible that at least the first 35 digits of c_{300} are correct:

$$c = 1.18610755201970969274641959912948880\dots$$

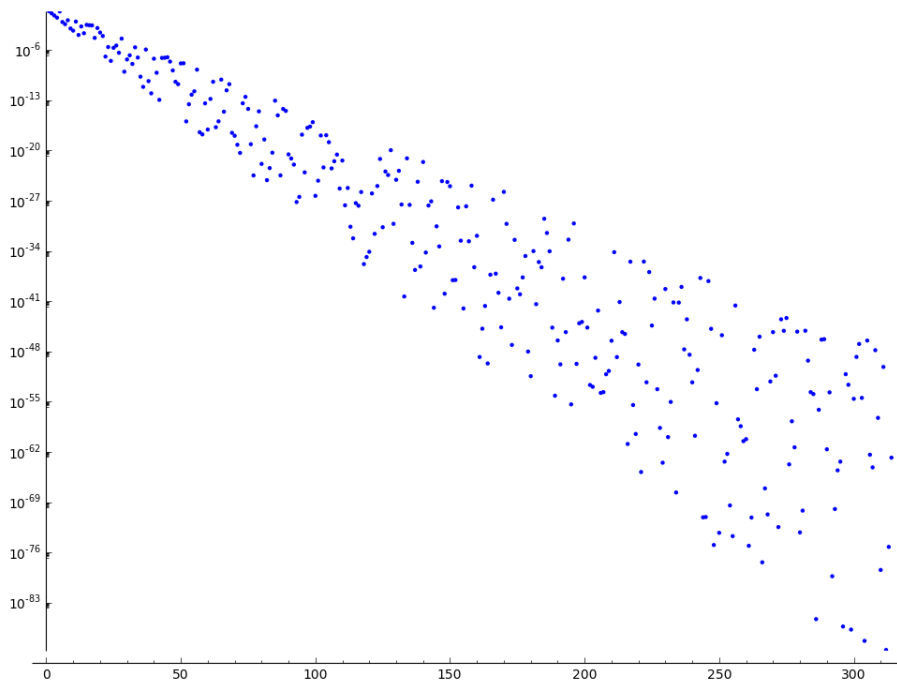


Figure 1: Logarithmic plot of $(|c_{n+1} - c_n|)_{n \geq 1}$ for $n < 315$.

4 Appendix

A table of the first 300 values of the sequence b .

n	b_n
1	2
2	5
3	11
4	23
5	47
6	79
7	157
8	313
9	619
10	1237
11	2473
12	4909
13	9817
14	19603
15	39199
16	78193
17	156019
18	311347
19	622669
20	1244149
21	2487739
22	4975111
23	9950221
24	19900399
25	39800797
26	79601461
27	159202369
28	318404629
29	636788881
30	1273577761
31	2547155419
32	5094310069
33	10188620041
34	20377200079
35	40754397121
36	81508794229
37	163017588457
38	326034863503
39	652069726981
40	1304139453961
41	2608278775139

n	b_n
42	5216557547329
43	10433115094657
44	20866228823311
45	41732454755713
46	83464902745219
47	166929802112809
48	333859603830469
49	667719207640333
50	1335438415261387
51	2670876800596783
52	5341753538059891
53	10683507076119781
54	21367014152239069
55	42734028304456861
56	85468056608789419
57	170936112956422291
58	341872225912844581
59	683744451825689161
60	1367488903651336099
61	2734977807302672179
62	5469955614604651421
63	10939911228867814129
64	21879822457735627939
65	43759644915471251671
66	87519289825471433329
67	175038579650942495623
68	350077159301184925933
69	700154318592312818233
70	1400380637184625633099
71	2800617274369251263599
72	5601234548738502526903
73	11202469097477005053763
74	22404938194953306492529
75	44809876389895526368621
76	89619752779790591190337
77	179239505559581182369501
78	35847901119162364739001
79	716958022238324715398701
80	1433916044476646061607879
81	2867832088953292123215433
82	5735664177906584244794839
83	11471328355813168489589671
84	22942656711626336979178663
85	45885313423252673958167347
86	91770626846498644550911261
87	183541253692997168082731737
88	367082507385994336165462423
89	734165014771984909918324339
90	1468330029543965802199019407
91	2936660059087931604391093729
92	5873320118175863208778430461
93	11746640236351726417555791877
94	23493280472703452835111583741
95	46986560945406905670223167347
96	93973121890813811214704529553
97	187946243781627622429407659893
98	375892487563255240394362905949
99	751784975126510467419366561643
100	1503569950253020823974756626427
101	3007139900506041647949513240631
102	6014279801012083295899023398461
103	12028559602024166579403443973367
104	24057119204048333158806008744053
105	4811423840809666626385803334291
106	96228476816193332516116637485681
107	19245695363238666503228132136167
108	384913907264773330064335369391327
109	769827814529546660126991628419769
110	1539655629059093320253983191729133
111	3079311258118186640506936021800541
112	6158622516236373281013872042403301
113	12317245032472746562027743464400949
114	24634490064945493124055486928796959
115	49268980129890986248110973857593677
116	98537960259781972496221947676827301
117	197075920519563944992443895320229633
118	394151841039127889984887785259447507
119	788303682078255779969775570518895013
120	1576607364156511559939551141037790007
121	3153214728313023119879102282075579809
122	6306429456626046239758204509009857359
123	12612858913252092479516409018019454749
124	25225717826504184959032815675304441663
125	50451435653008369918037625328347919813
126	100902871306016739836075250656678663449
127	201805742612033479672148495650051520389
128	403611485224066959344295693300427135063
129	807222970448133918681164641485718226971
130	1614445940896267837362329282970615449851
131	3228891881792535674724656302968912476923
132	6457783763585071349449232530051367454757
133	12915567527170142698898465056894676674687
134	25831135054340285397796930113789353349373
135	51662270108680570795559092648964890764943
136	103324540217361141591118185274937075902777
137	206649080434722283182236370549873916636879
138	413298160869444566364472741099747833194221
139	826596321738889132728945179098738031574433

n	b_n
140	1653192643477778265457890358197476062179889
141	3306385286955556530915092188941585586136419
142	6612770573911113061830184377883170833420293
143	13225541147822226123660368753373494242944751
144	26451082295644452247320737488693714733719843
145	52902164591288904494641474977387429467439631
146	105804329182577808989282949954750676440829051
147	21160865836515561797856589990501276468133839
148	423217316730311235957131610262875945813279907
149	846434633460622471914263220525751891626480523
150	169286926692124494382852586213247758885348913
151	3385738533842489887657051425372313993469761381
152	6771477067684979775314102850744627986890683163
153	13542954135369959550628205701489255973677941979
154	27085908270739919101256411400290734123962938413
155	54171816541479838202512822800581345027613472609
156	108343633082959676405025645601162690055226855289
157	216687266165919352810051291171823048574660844837
158	433374532331838705620102582343645277142077112269
159	866749064663677411240205073008093194936538909311
160	1733498129327354822480410146016186389075140132929
161	3466996258654709644960820292032334698932001753131
162	6933992517309419289921640584064669397864003506261
163	13867985034618838579843281168129338795728006995091
164	27735970069237677159686562336258677591455963272631
165	55471940138475354319373124672517355182911926545261
166	110943880276950708638746249345034710361207948122501
167	221887760553901417277492498430626236480126605967189
168	443775521107802834554984996861252472934553043584927
169	887551042215605669109969993722504945868994693217131
170	1775102084431211338219939987445000981737989382946869
171	3550204168862422676439879924085462500286942930288747
172	7100408337724845352879759848167343485740169237218921
173	14200816675449690705759519696334686971480067965888653
174	28401633350899381411519039392669373942960135931581573
175	56803266701798762823038078785338579861311415592325389
176	113606533403597525646076157570677159722564478499636719
177	227213066807195051292152315141354319445111315724194669
178	454426133614390102584304630282708638882295687973841727
179	908852267228780205168609260565417262590301863599776453
180	1817704534457560410337218521130834525180603727198057459
181	363540968915120820674437042261669050361207454396113801
182	7270818137830241641348874084523337509475789397803130441
183	14541636275660483282697748169046675018951530701409081869
184	29083272515320966565395496338093349967298437421522015171
185	5816654510264193313079099267618669990825568844559989813
186	116333090205283866261581985352073721705987641170514588653
187	232666180410567732523163970704141171306011023637359104221
188	465332360821135465046327941408282306743300539297354701951
189	930664721642270930092655882816564613486601076933474162623
190	186132944328454186018531176563312922697320215386948324159
191	372265888656908372037062353126625845394640430762422306589
192	7445317773138167440741247062532516907892808615252744279599
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194	297812710925526697629649882501300676312270827358941313232423
195	59562542185105339525929976500259945750650846841758045509521
196	119125084370210679051859953000519891501301693683516091044731
197	238250168740421358103719906000902738954784077869126178024667
198	476500337480842716207439812001805477909568155738244817036261
199	953000674961685432414879624003610955819136304707961495391653
200	1906001349923370864829759248007221911638272588458529407007529
201	381200269984674172965951849601444382320928664835356055118583
202	7624005399693483459319036992028887646418573282948868304030099
203	15248010799386966918638073984057775292837146565897736331682923
204	30496021598773933837276147968115550585674293131795472351055147
205	60992043197547867674552295936231101171348586263584096828788511
206	12198408639509573534910459187246220234269711021829987972351213
207	24396817279019147069820918374492440468539424043659975593840689
208	487936345580382941396418367489848809370788484087319950291479491
209	975872691160765882792836734979697618741576968174639343961587663
210	1951745382321531765585673469959395237483153936349275530031214669
211	390349076464306353117134693991879047496630787259504244102757267
212	7806981529286127062342693879837580500455100912017621363963610833
213	15613963058572254124685387759675161000910201824033072219830085753
214	31227926117144508249370775519350322001820199871088658892901095259
215	62455852234289016498741551038700644003640399716432976019129441607
216	124911704468578032997483102077401288007280799403812791749844240237
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219	999293635748624263979864816619210301354239043792462772846357491531
220	1998587271497248527959729633238420602708478087584925545692709516887
221	399717454299449705919459266476841205416956175169800358829592511173
222	7994349085988994111838918532953682410833912350339600717659185022237
223	15988698171977988223677837065907364776826709209580242927075183971817
224	31977396343955976447355674131814729553653418419160484477099561947927
225	63954792687911952894711348263629459101052517387822722335599730888349
226	127909585375823905789422696527258918202105034332722390596913787068743
227	255819170751647811578845393054517836404205081978541845601590856295469
228	511638341503295623157690786109035672808410163957083688799529974591459
229	1023276683006591246315381572218071345616820327914167377599040431771409
230	2046553366013182492630763144436142691233640655828334755198080862990191
231	4093106732026364985261526288872285382465562483910334253800762395808969
232	818621346405272997052305257744570764931124967820668507601516652648309
233	16372426928105459941046105155489141529862249935641337013906683812243789
234	32744853856210919882092210310978283059724312690059928494601616081212307
235	65489707712421839764184420621956566119448625380119856989203232162423353
236	130979415424843679528368841243913132228896539702802219997166011588487397
237	2619588308496873590567368248782624477576831535557383623761422136906367

n	b_n
238	523917661699374718113475364975652528955153663070250585777070745250516679
239	1047835323398749436226950729951305057910307299550978669347757831020260369
240	2095670646797498872453901459902610115820614599101329655268072774674170577
241	4191341293594997744907802919805220231641229198202659133867855734894926187
242	8382682587189995489815605839610440463282458396405318267735698648178758789
243	16765365174379990979631211679220880926564916792810599676059931222142134879
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246	134122921395039927837049693433767047105570422927247806940219528856401551609
247	268245842790079855674099386867534094819666048417592945679659126255260488409
248	53649168558015971134819877373506818963932096191959251949099018888780931821
249	107298337116031942269639754747013637927866419238391850389981803777561863641
250	2145966742320638845392795094940272758557328384767837007685454667230890261093
251	429193348464127769078559018988054551711465676953567401537090933461780521987
252	858386696928255538157118037976109103422931353773951463593097077679883921701
253	17167733938565110763142360759522182068458627075479029271861941546980322680503
254	34335467877130221526284721519044364136917254150958058543723882936216190884029
255	68670935754260443052569443038088728273834508301916117087447765872432361006359
256	1373418715085208861051388860761774565476690166038323241748955317448667422010521
257	27468374301704177221027777215235491309533745422714595507737801831837774828023
258	54936748603408354442055544304709826190674908454291910154592365587823704812019
259	109873497206816708884111088609419652381349816908583820309148899324887769356741
260	2197469944136334177682222177218839304762699633817167640618297201774151801350409
261	439493988272668353644443544376786095253992676343352812365921363643852634624137
262	8789879776545336710728888708875357219050798535266870562473184272728770565257923
263	1757975953090673421457777417750714438101597070537341124946368545454102268971
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